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QCD临界点附近的动力学临界涨落

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摘要 探寻量子色动力学（Quantum Chromodynamics，QCD）相图的临界点是相对论重离子碰撞实验低能量扫描（Beam Energy Scan program in Relativistic Heavy-Ion Collisions，RHIC-BES）项目的重要实验目标。初步的实验探测发现了净质子数涨落随着对撞能量的非单调行为，定性上符合基于静态模型的理论预言，这暗示着QCD临界点的存在。由于相对论重离子碰撞是一个高速膨胀的体系，动力学效应能显著地改变QCD临界点附近的临界涨落。为最终确定QCD临界点的存在以及研究QCD在有限温有限密区域的相结构，人们发展了一系列QCD临界点附近的动力学模型。本文回顾了近年来关于寻找QCD临界点在实验测量和理论模型的进展，着重强调在临界点及一级相变区域动力学模型的发展和挑战。

关键词 QCD相图及相变，相对论重离子碰撞低能量扫描，动力学模型
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Critical dynamical fluctuations near the QCD critical point

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Abstract The exploration of the critical point on the QCD (Quantum Chromodynamics) phase diagram is one of the most important goals of the beam energy scan program in relativistic heavy-ion collisions (RHIC-BES). Preliminary experimental measurement observed the non-monotonic behavior of net-proton fluctuations as a function of collision energy, which qualitatively agrees with the prediction of the static theoretical models and this hints the existence of the QCD critical point. The system created in heavy-ion collision is highly expanding system with which the dynamical effects dramatically modify the critical fluctuations near the QCD critical point. To confirm the existence of QCD critical point and study the phase structure of QCD system at finite temperature and finite density region, a series of dynamical models near the QCD critical point has been developed. This paper reviews the recent developments related to the exploration of the QCD critical point from experimental and theoretical viewpoints. In particular, we emphasize on the developments and challenges of the dynamical model near the QCD critical point and the first-order phase transition.

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量子色动力学(Quantum Chromodynamics,QCD)[1]是描述不同能标下强相互作用物质的理论。在高能标区域,QCD很好地解释了质子碰撞实验[2]所发现的渐进自由行为[3-4];在低能标区域,实验上关于真空对称性及其破缺行为的结果很好地支持了QCD的理论预言[5-6]。有限温有限密下的强相互作用理论和模型计算预言了核物质系统存在着丰富的相结构。具体说来,在低温低密区域,强相互作用系统由重子和介子等强子组成。带有色荷夸克和胶子囚禁在强子内部,形成色中性的强子,因此,系统处于禁闭相(Confinement Phase)。同时,手征对称性发生破缺,系统也处于手征对称性破缺相(Chiral Symmetry Breaking Phase)。在高温高密区域,带有色荷的夸克和胶子从强子中解禁闭,形成夸克胶子等离子体(Quark-Gluon Plasma,QGP),故强相互作用系统处于解禁闭相(Deconfinement Phase)。此时,手征对称性恢复,故系统处于手征恢复相(Chiral Symmetry Restoration Phase)。在自然界中,宇宙大爆炸早期和中子星等致密星体内部可能存在QGP。在实验室中,极端条件下的相对论重离子碰撞可产生QGP,并以此研究强相互作用物质的相图和相变[7-10]。目前,运行的相对论重离子碰撞实验包括美国布鲁克海文国家实验室RHIC(Relativistic Heavy Ion Collider)、欧洲核子中心LHC(Large Hadron Collider)的重离子碰撞项目。此外,在建的实验装置还有德国的FAIR(Facility for Antiproton and Ion Research)[11]、俄罗斯的NICA(Nuclotron-based Ion Collider fAcility)[12]、中国的HIAF(High Intensity heavy ion Accelerator Facility)[13],以及日本的J-PARC(Japan Proton Accelerator Research Complex)[14]等项目。经过几十年的研究探索,基于相对论重离子碰撞相关的研究有了长足的发展,使人们对强相互作用物质的理论有了较深的理解。

1 QCD相图与较低碰撞能区的相对论重离子碰撞

强相互作用物质体系由QCD理论描述,其相变和相图亦称为QCD相变和相图。通常QCD相图在温度和重子化学势张成的平面中表示。在低化学势μ区域,格点QCD的计算表明,在化学势$\mu = 0$ MeV温度$T \simeq 155$ MeV[15-17]区域从强子相连续地过渡到QGP相[18-21],即连续过渡(Crossover);在高化学势区域,基于QCD的有效模型,如泛函重整化群理论(Functional Renormalization Group,fRG)、戴森-施温格方程方法(Dyson-Schwinger Equations,DSE)等,预言强子相和QGP相之间的相变为一级相变(First order phase transition)[22-25]。自然地,人们预言在相图中的连续过渡和一级相变之间存在一级相变的终点,即临界点(Critical point)[26-27]。在有限化学势($\mu > 0$)区域,格点QCD存在符号问题[28],即积分测度变成复数。为解决这一问题,很多方法被发展起来,如泰勒展开[29-30]、重分配权重[31]、虚化学势[32]等。最近的格点计算结果给出临界点位置可能在$T <$ 140 MeV, $\mu_B >$ 300 MeV[33]。另一方面,基于QCD的连续场论模型,如fRG给出了临界点在QCD相图上的可能区域为:450 MeV $\leqslant \mu_B \leqslant$ 650 MeV[25]。然而,不同有效模型预言的临界点位置明显地依赖于模型和参数的选取[27]。因此,QCD临界点是否存在及可能的位置还是很不确定,亟待人们对相关问题更加深入地研究。

在相对论重离子碰撞实验中,重子阻塞效应(Baryon stopping effect)使得所产生的热密核物质的重子化学势随着对撞能量的降低而增大[34]。目前,在相对论重离子碰撞实验能量扫描(Beam Energy Scan program in Relativistic Heavy-Ion Collisions,RHIC-BES)项目的Au+Au对撞的能量范围为200~7.7 AGeV[35-36]。基于统计模型[34]的计算表明,其对应的温度T区间为100~180 MeV,重子化学势μ为50~600 MeV。这个范围包含了多个模型所预言的QCD临界点存在的可能区域。因此,人们可以通过改变相对论重离子对撞的能量$\sqrt{s_{NN}}$来探究QCD相图及相变临界点。目前,第一阶段的能量束流扫描(BES-I)[37-38]已完成。通过分析末态净质子数的高阶累积量,人们发现,在能量小于39 GeV区域净质子四阶累积量的比值(Kurtosis)显著地偏离背景涨落的基线,且随着对撞能量呈现出非单调行为。这在定性上与平衡态临界涨落模型[39-40]给出的预言吻合,预示着发现QCD临界点的可能性。经过进一步升级,第二阶段的能量束流扫描项目(BES-II)[41-42]已经于2019年初开始运行,较为完整的实验测量结果也将于最近公布。这亟待人们在理论上构造并发展模型[43-46],以解释相应的实验数据并预言新的观测量。

2　QCD临界涨落与实验信号

在QCD相图上，临界点是一级相变线的终点，其最重要的特征是关联长度发散。系统关联长度发散会导致系统在临界点附近的一系列的独特性质，如涨落发散、奇异性、普适性以及临界慢化等。其中，最著名的例子是临界乳光现象：即水的温度和压强接近临界点时，系统的关联长度显著增加并接近照射光的波长，导致光被散射，进而形成临界乳光。在相对论重离子碰撞实验中，人们无法直接探测实验中产生的火球性质，只能在末态观测量中寻找可能遗留的QCD临界信号。本节将简要介绍相对论重离子碰撞中与QCD临界涨落相关的实验信号。

2.1　逐事件粒子数涨落

描述手征相变的序参量可以为手征凝聚$\langle\bar{\psi}\psi\rangle$（通常定义为$\sigma$场：$\sigma\equiv\langle\bar{\psi}\psi\rangle$）。在高温高密情况下，强相互作用物质会发生相变形成QGP，其手征对称性恢复，手征凝聚平均值为零（$\langle\bar{\psi}\psi\rangle\simeq 0$）；在相变温度以下，QGP转变为强子物质，其手征对称性破缺，手征凝聚平均值不为零（$\langle\bar{\psi}\psi\rangle\neq 0$）。在临界点附近，表征手征凝聚的$\sigma$场会发生长程关联。而$\sigma$场与末态粒子的耦合$\sigma NN$会导致末态粒子的长程关联以及大的逐事件涨落[47–51]。随着对撞能量的变化，该粒子数涨落会呈现出非单调的行为[39–40]。由于守恒量与理论预言（如格点QCD[52–57]和强子共振气体模型（HRG）模型[58–60]）的磁化率直接相关且对关联长度ξ很敏感，一般认为系统守恒量，如重子数（B）、电荷数（Q）与奇异数（S）的涨落是寻找QCD临界点的灵敏探针[61–62]。此外，由于手征临界效应不会导致奇异的同位旋磁化率，故可以用净质子数涨落作为净重子数涨落的近似[49–51]。基于此，RHIC-BES实验通过测量净质子数的涨落来探寻QCD的临界点。由于QGP火球的有限时间和有限体积效应，关联长度被限制在2~3 fm[63–64]，这使得仅使用逐事件粒子数涨落的二阶累积量探测临界信息变得困难。因此，人们提出高阶累积量对关联长度更加敏感[61–62]，其中的四阶累积量在临界区域会有符号的变化[39]。此外，长程关联的效应会使得累积量随着实验探测器的接收度变大而变大[65–67]，即接收度依赖行为。

实验上，RHIC-BES第一阶段项目探测了能量$\sqrt{s}$为7.7 GeV、11.5 GeV、14.5 GeV、19.6 GeV、27 GeV、39 GeV、54.4 GeV和200 GeV下Au+Au碰撞的净质子数涨落（动量范围为$|y|<0.5$，$0.4<p_T<2$ GeV）[37,68–70]。结果表明：随着对撞能量的降低，净质子数涨落四阶累积量比值$\kappa\sigma^2$先降低后上升，在对撞能量约为19.6 GeV附近形成最小值。近期的HADES（High Acceptance DiElecton Spectrometer）实验探测了对撞能量为2.4 GeV的Au+Au净质子数涨落，相应的$\kappa\sigma^2$为负值[71]。综合不同的实验测量结果，净质子数涨落的四阶累积量的非单调行为与平衡态临界涨落的理论模型预言大体一致[39–40]。随着探测器的接收度增大，净质子数涨落也快速地变化，这与理论[65–67,72–74]预言的接收度依赖行为一致。初步的实验探测与理论预言的吻合暗示临界点存在的可能性。然而，三阶累积量的实验结果与理论预言不符[66]，这需要进一步考虑动力学临界涨落的影响，相关研究进展将在§3讨论。在小于7.7 GeV的低能区域的行为需要进一步的探索研究，一方面该区域可能存在一级相变，另一方面，该区域是否生成QGP还有很大的争议。此外在低对撞能量区域，探测数据的误差还很大，这需要在BES第二阶段和固定靶实验中更大的统计量和更大探测器接收度的实验结果[41–42]。

2.2　轻核产额比值

在临界点附近，组成轻核的核子有很大涨落。相关研究表明，轻核产额与中子密度坐标空间涨落相关[75–76]。最近的STAR实验组系统地探测了在RHIC-BES项目中的轻核产额比值$N_t N_p/N_d^2$。在中心快度区（$|y|<0.5$）的中心碰撞中发现了轻核产额比值随着对撞能量的非单调性[77]。由于较小的结合能，轻核产额受到很多因素的影响。现有基于流体力学等并合模型的计算无法很好地拟合实验数据[78–82]，此外，一些数值模型也研究了临界点附近相互作用对轻核产额的影响[83–87]，同时，人们也考虑了末态共振态等效应[88]。为确定实验中的非单调行为是否为临界点的信号，理论上需要精细地研究各种因素对轻核产额的影响以排除背景因素的污染，故人们进一步细致地研究了并合模型（Coalescence model）中粒子相空间分布的影响，发现了轻核产额比值对高阶相空间分布累积量的敏感性[89]。在引入了静态临界关联的贡献后，通过构造轻核产额一系列组合压低背景效应影响，使其与关联长度成正比[90]。基于此，新的轻核比值被提出来，且预言了临界区域双峰结构。这需要后续更高统计量及包含更多核子组成的轻核的实验数据，以确定这类特征的临界点信号。

3　QCD临界点附近的动力学模型

由于相对论重离子碰撞实验中产生的火球是一个高速膨胀的系统，这使得系统处于临界区域的时

间变得有限。由于临界慢化效应，系统的关联长度等物理量来不及跟上平衡态关联长度的增大，对应的涨落称为非平衡态临界涨落。为考虑非平衡效应，人们需要构造相应的动力学临界涨落模型。对于临界点附近的系统，其动力学演化由动力学普适类决定。动力学普适类是根据系统序参量的对称性、维度、守恒量的守恒方程以及守恒量之间的泊松括号决定[91]。人们[92]认为有限温的QCD系统属于动力学普适类中的model H，即包含了守恒序参量，守恒的动量密度和它们之间的泊松括号组成的动力学系统。由于相对论重离子碰撞中model H在数值上模拟的复杂性，作为model H的替代，人们发展了基于守恒和非守恒序参量演化的动力学模型以及基于流体力学与临界自由度耦合的临界动力学模型。本节将概述相关研究的进展，同时简要介绍临界点附近的特征动力学行为，如动力学普适标度律等。

其中一个简单模型是仅考虑非守恒序参量场的演化，即Model A。由于序参量场是非守恒量，其运动方程为弛豫方程，形式上可以写成Langevin方程[93–95]或Fokker-Planck方程[96–97]。除了序参量场，QGP系统还需要考虑其他自由度。在Model A中，序参量场以外的自由度处理成热浴与σ相互作用，在Langevin方程中表现为噪声项。早期的研究表明临界慢化效应不仅压低了关联长度的增大行为[63–64]，而且显著地改变各阶矩的时间演化行为，与平衡态临界涨落相比其符号可能发生翻转[93–97]。

一种简单的耦合方式仅需要考虑序参量场与守恒荷间的耦合。由于在临界点附近守恒荷变为更慢的模式[92]，因此，系统的演化只需处理其守恒模式的随机扩散的方程，并将其他的模式处理成热浴，这就是Model B[72–74, 98]。基于Model B，人们[72–73]发现了通常的临界慢化现象。同时，守恒荷的临界动力学行为的一个突出特征是在临界点附近关联长度的增大和体系扩散效应的相互竞争，进而导致守恒荷粒子数涨落随着接收度的增大出现非单调行为[72–73]。

较为复杂的耦合方式是将序参量直接处理成流体力学模型额外的自由度，并作为源项耦合到流体力学模型中，即非平衡手征流体力学（Non-equilibrium chiral Fluid Dynamics，NχFD）[99–104]。这里对序参量场的处理类似于Model A，其演化满足Langevin方程且有效势从基于QCD的有效模型得到，如(P)QM模型[105]、线性σ模型[106–107]等。人们已经发展了非平衡态手征流体力学的数值模拟以研究QCD相变附近的一系列动力学行为，其中，发现净质子数涨落在临界点附近的方差和kurtosis相比于连续过渡区域有较大的增强[99–100]。

为了更为自洽地考虑序参量与流体方程的耦合，人们考虑在临界点附近慢模式演化，以研究流体背景下临界点相关模式的动力学行为，即hydro+[108]。因为是系统最慢的模式，熵除以重子数s/n的两点函数在hydro+被选取为额外的自由度。为了自洽地耦合慢模式自由度，相应的状态方程和输运系数也需要修正。对简单的流体系统，如快度无关的径向流膨胀[109]或者Gubser流[110]中，hydro+的数值模拟表明，慢模式对流体的影响几乎可以忽略，这种非平衡效应对熵只有0.1%的贡献。相应地，人们也分析了慢模式自由度的引入对末态freezeout修正[111]和声速的改变[112]。除了由两点函数引入的慢模式影响，其他的慢模式影响（如hydro++[113–114]）也正在发展中。

在临界点附近，序参量场的涨落会传递到流体系统，进而导致流体的涨落，即涨落流体力学模型（Hydrodynamics fluctuation）。为了定量描述实验数据（如净质子数涨落），一种自然的方法是仅考虑包含随机性的相对论流体力学。作为一种有效模型，流体力学模型处理的自由度为守恒量，如能量动量张量$T^{\mu\nu}$和重子流N^{μ}。在临界点附近，作为热浴的手征序参量场通过σNN耦合与流体相互作用，其贡献可以写成流体力学方程中的噪声项[115]。其中的噪声项由涨落-耗散定理确定，作为一阶近似可以处理成白噪声。涨落流体力学模型非相对论形式最初由Landau等[116–117]提出，后来被推广到相对论形式[115]。原则上，在输入包含临界点的状态方程和输运系数的涨落流体力学模型中，人们可以细致地研究临界点附近的动力临界涨落，并给出其对末态观测量的预言。例如，基于0+1维boost invariant膨胀的背景，解析的计算[118]表明，关联函数在QCD临界点附近因为热导率的增大而有较大的增强。然而，在实际的数值模拟中，需要考虑依赖于空间的噪声效应[119–121]以及对数值格子大小的依赖问题，相应的状态方程和输运系数需要考虑重整化[122–123]。相关的研究仍然处于发展中[124–128]。

由于涨落流体力学模型在数值模拟上的困难，人们发展了确定性的流体力学方程，以解析计算并演化涨落的两点关联函数[129–132]，即流体运动学（Hydrodynamics-kinetic）。流体运动学自然地包括了已经被重整化的输运系数和状态方程，且吸收了涨落流体力学模型在数值模拟中的格距依赖问题。流体运动学能重复出涨落流体力学模型的长尾效应（Long-time tails）[123,133–135]并估算出临界涨落的相关尺度[132]，以及在连续过渡区域对π介子的影响[136–138]。除了两点关联函数，高阶关联函数（如三

点和四点关联函数)的流体运动学方程也被发展了起来[139-140]。

除了构建临界点附近的动力学模型,作为其输入,包含QCD临界点的状态方程也是非常重要的研究内容。如上所述,在有限化学势区域格点QCD计算存在符号问题,同时有效模型预言的状态方程依赖参数选取。另一方面,由于有限温QCD体系与三维Ising模型属于同一静态普适类[141-144],通过非普适的映射关系[64,145-146],人们构造了包含临界点的参数化状态方程[147-151]。其核心思想是状态方程的背景贡献由格点QCD在化学势$\mu = 0$区域延拓给出,拼接上三维Ising模型的临界部分贡献。原则上,可以通过在流体力学模型计算输入此类状态方程并与末态观测量比较给出该状态方程的一系列参数,大致确定QCD临界点的位置等信息。为了确定模型中的一系列参数,人们需要实验上探测一系列信号,在构造精细的理论模型的基础上,利用贝叶斯等方法约束出相关参数。相关研究仍处于初始阶段,需要后续进一步的发展完善。

描述相对论重离子碰撞实验的另一种常用的动力学模型为基于玻尔兹曼方程的微观输运模型,如多相输运模型(A Multiphase Transport model, AMPT)[152-153]、极端相对论量子分子动力模型(Ultra-relativistic Quantum Molecular Dynamics model, UrQMD)[154-156]及喷注AA微观输运模型(Jet AA Microscopic transport, JAM)[157]等。基于这类微观模型,通过引入临界点附近的状态方程修改微观粒子之间的相互作用,人们研究了相对论重离子碰撞实验中一系列的观测量,如质子数涨落[158-163]、集体流[164-165]、相图中的演化轨迹[166]及HBT关联[167]等。与宏观的流体力学模型类似,临界点附近的微观输运模型也需要考虑临界涨落的贡献。基于玻尔兹曼方程,文献[168-170]将微观粒子相空间分布线性展开,发展了微观输运模型的涨落与关联函数满足的动力学方程。进一步地,文献[171]也将序参量场与玻尔兹曼方程耦合,分析了在强子相临界涨落的演化。

临界点的一个重要的特征行为是普适标度律。由于在临界点附近关联长度发散,系统没有特征尺度,因此在尺度变换下,系统会有自相似行为。平衡态系统的临界普适类由序参量、对称性和系统维度等决定。属于同一普适类的不同系统,其临界行为是相同的且由临界指数确定。通过对称性分析,人们通常认为QCD临界点和三维Ising模型属于同一普适类[172-173]。对于相对论重离子碰撞实验中的火球系统,需要考虑有限体积的效应,即有限体积标度律[174-175]。另一方面,临界点附近的普适标度行为在动力学过程中需要推广为动力学标度律,即引入额外的临界指数z。动力学标度律是系统的膨胀效应与弛豫效应相互作用的结果。临界点附近的临界慢化效应使得弛豫效应增大,但膨胀效应使得系统无法弛豫到平衡状态,临界模式变得非平衡,非平衡的关联长度无法跟上增大的平衡关联长度。因此,系统的特征尺度由非平衡关联长度决定,这种非平衡关联长度被定义为Kibble-Zurek尺度[176-177],由膨胀速率和弛豫速率相等时确定。在相对论重离子碰撞中,人们[97]发现根据Kibble-Zurek机制,可以在临界点附近构造一些不依赖于系统演化轨迹等因素的普适物理量。同时,基于Model A的不守恒序参量[94,97]和Model B演化守恒荷的演化方程[74,132],同样可以构造物理量的普适行为。由于这种普适行为,净质子数涨落的高阶累积量等物理量在重新定义下,高度震荡的行为被压低,并近似地收敛到一条曲线中[94]。

4 QCD一级相变及动力学模型

与临界点类似,系统在一级相变附近存在着丰富的物理过程,如过冷(Supper cooling)、过热(Supper heating)过程等。一级相变最重要的特征之一是系统的不稳定性:由于自由能存在亚稳态,当系统演化到一级相变区域,系统将分离成两个共存的相。根据相变理论[178],这种不稳定过程可以分为两种不同的过程:成核(Nucleation)和Spinodal分解(Spinodal decomposition)。成核作为一种非线性不稳定性,需要稳定相形成足够大的核以越过势能中稳定态和亚稳态之间的势垒。一级相变的经典成核理论在20世纪30年代已经被建立起来[178],并经过了一系列的发展[179-184]。作为理论的核心,系统的成核速率描述的是单位时间和单位体积内从亚稳态的衰变速率:$\Gamma \sim e^{-\Delta\mathcal{F}_c}$,其主要决定因素为$\Delta\mathcal{F}_c = \Delta\mathcal{F}(R_c)$,这里的$R_c$为成核的临界半径。与成核过程不同,Spinodal分解是一种线性不稳定性。在系统处于自由能的曲率为负的时候,小的序参量涨落能够快速且在全空间均匀地增大,其理论基础是在化学冶金领域广泛运用的Cahn-Hilliard方程[185]。

通过推广经典的成核理论,人们[186-194]在相对论重离子碰撞中估计了QGP中的强子气泡的成核率,研究表明强子气泡形成率和过冷(或再热过程)受到很多因素的影响,比如自由能中稳定态和亚稳态之间的势垒高度,输运系数以及膨胀速率等。基于上述动力学模型,如Model A[93]和NχFD[100-104],人们也研究了一级相变附近的动力学行为,其中得到了过

冷和再热效应对末态观测量的影响。在高速膨胀的火球中，系统会进一步地演化进入Spinodal分解区域。因此，利用耗散流体力学模型且输入包含共存相的状态方程，人们[195–199]研究了Spinodal分解过程，发现了禁闭相和解禁闭相的分离及其实验观测效应，如密度分布矩在Spinodal分解区域的增强效应。

尽管相对论重离子碰撞中一级相变附近的动力学研究取得了一些进展[200–201]，一级相变的复杂性使得人们对相关物理过程的理解还很不足。理论上，由于系统在一级相变附近的非平衡效应，人们对于流体力学的适用性还很不清楚。唯象上，研究相对论重离子碰撞实验中一级相变的不稳定性还有一定困难。QCD相图中的一级相变可能对应着相对论重离子碰撞实验的低对撞能量区域。随着对撞能量的降低，碰撞的两个原子核需要更长的时间穿过对方，即系统的初始条件变得更加重要[202–203]。同时碰撞火球的初始温度进一步降低，对应着由流体描述的QGP系统占整个演化比例降低。当能量扫描实验降低到特定对撞能量，系统不再形成近平衡的QGP。例如，STAR实验的测量发现对撞能量小于3 GeV的椭圆流v_2变负[204]；另外，HADES实验的测量结果发现，2.4 GeV的净质子数$\kappa\sigma^2$为负值[71]。这一系列信号暗示着在低对撞能量区域系统的自由度可能为强子。在此区域，人们需要进一步发展基于强子自由度的相变附近动力学模型。另一方面，在高化学势区域的状态方程也需要进一步的发展。如前所述，在高化学势区域的格点QCD的符号问题[28]，以及基于QCD的有效模型的参数依赖问题[27]等。这亟待人们在理论上和唯象上对一级相变附近的物理过程及可能的实验效应做更加深入的研究。

5 总结与展望

相对论重离子碰撞实验是研究极端条件下核物质的重要手段，是当前高能核物理的研究热点[205–218]。对QCD相图的研究是相对论重离子碰撞实验的重要研究目标，通过改变对撞能量能够扫描QCD相图很大的区域。净质子数涨落及轻核比值等随着对撞能量有非单调的行为；进一步地，考虑到对撞实验中迅速演化的火球系统，人们发展了一系列非平衡态临界涨落模型。基于理论上预言临界点附近的临界涨落对末态观测量的影响以及RHIC上BES项目对净质子数涨落的探测，初步结果预示着QCD临界点的存在。

为进一步确定QCD临界点的存在及其可能的位置，需要发展更为完善的动力学模型。由于相对论流体力学在RHIC高能区域以及LHC能标的巨大成功，为系统研究临界点附近的动力学过程，需要进一步发展基于流体力学的动力学临界涨落模型。例如考虑不同慢模式自由度的hydro++模型，并推广到三点和四点函数组成的慢模式自由度等。除此之外，随着BES能标的降低，初始条件以及末态演化在复合演化过程中占据更大的比重，因此较为完整地考虑各个阶段的效应对定量解释实验数据有着十分重要的意义。同时，在BES能区，动力学方程的输入量，如状态方程，输运系数等也亟待更加准确的结果。另一方面，除了完善唯象动力学模型以定量地解释实验数据，构造几个对QCD临界点敏感的实验信号也有助于确定临界点是否存在。由于相对论重离子碰撞实验中产生的火球是一个复杂的体系，定量地符合实验结果会有较大难度。通过构造对临界点敏感观测量以压低背景因素的影响并预言临界点导致的特征行为，理论与实验的吻合将对临界点的确定提供重要的支撑。

实验上，RHIC上BES二期项目的测量结果将提供更高精度更大接收度的净质子数涨落数据，以降低各阶累计量在20 GeV以下的统计误差，最终确定净质子数涨落是否存在理论上预言的非单调行为。同时BES二期项目也运行了更低能量的固定靶实验，这为人们研究更大的QCD相图区域提供了可能。通过与更加完善的理论模型预言相互比较，使得人们可以深入研究高温高密下QCD相结构。

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